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GENERAL SOLUTION OF DIFFUSION PROCESSES IN SOLID–LIQUID EXTRACTION

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Abstract—An analytical solution is found for the concentration distributions in q different particles of arbitrary finite geometry and initial conditions to be extracted in batch of finite volume. The variation of concentration of the surrounding liquid is coupled with separation of substance from the porous particles at the mass balance conditions for the liquid phase. The solution obtained contains in itself as a very special case the problem of the periodical, co- and counter-current extraction from one-dimensional particles (plate, cylinder and sphere) [1, 2, 8–10], the problem of the temperature distributions in solid particle heated by gases at co- and counter-current [3, 5] and the problem of heating a body in a bounded volume of a well mixing liquid [4, 7].

NOMENCLATURE

A_m, B_m, C_m, constant boundary coefficients;
$f_m(M), f_m(x), f_m(\xi)$, initial distribution functions;
$w_m(M), w_m(x), k_m(M), k_m(x), P_m(M,\tau), P_m(x,\tau)$,
$Q(\tau)$, prescribed functions;
i, 1, 2, 3, ...;
m, 1, 2, ..., q;
M, point in V_m;
N, point on S_m;
n, outward normal of S_m;
S_m, boundary of V_m;
p, Laplace transform parameter;
φ_0, θ_0, initial liquid potential;
$\varphi(\tau), \theta_f(Fo)$, unsteady liquid potential;
$\psi_{mi}(M), \psi_{mi}(x)$, eigenfunctions;
μ_i, eigenvalues;
Γ_m, formfactor of the particles equal to 0, 1 and 2 for plate, cylinder and sphere respectively;
R_m, characteristic length of the particles (half the thickness of the plate or the radius for the cylinder and the sphere);
R, average size of the particles;
D_m, D, diffusion and average diffusion coefficient of particles;
ξ, $= r/R_m$, non-dimensional co-ordinate;
Fo, $= D\tau/R^2$, Fourier number;
ω_m^2, $= \dfrac{DR_m^2}{D_m R^2}$, non-dimensional numbers;
Bi_m, $= \rho_m R_m/D_m$, Biot number where β_m is the coefficient of mass transfer between the fluid and the particles;
K_m, $= \dfrac{\Gamma_m+1}{\omega_m^2}\dfrac{V}{V_f}$, non-dimensional numbers, where V_f and V are the volumes of the extracting liquid and the liquid, filling the pores of all the particles;
$\theta_m(\xi, Fo)$, non-dimensional concentration of particles.

INTRODUCTION

The processes of extracting substance from solid porous media are widely disseminated. As an example one can consider the extraction of sugar from sugar white beet turnings, the extraction of plant oil from seeds and so on. In the production of mineral salts in chemical industry the first stage of the process is the lixivation, that is the extraction of the soluble components from the mineral raw materials by means of water or water salts. Analogous processes take place in extractive metallurgy, where for the dissolution weak solutions of acids or salts are used [1, 2].

All the above mentioned processes have a common mechanism of mass transfer from the inside of the porous body and are described by the well-known diffusion equation [1, 2]. The general solution obtained in [11] enables one to study the kinetics of the process if one assumes that the concentration of the extracting liquid is a preliminary given function of time.

But the preliminary assumption for the law of the variation of the concentration of the extracting liquid is not correct because the accumulation of substance in the extracting liquid is connected with the separation of substance from the porous particles. This is why one is to couple the variation of concentration of the surrounding liquid with the one of the porous particles through the differential equations of the mass balance. Such a method is used in [1–10] and others.

Detailed solutions for one-dimensional bodies (plate, cylinder and sphere) are given in [4, 6] where it is pointed out that the problem of determining the kinetics of the periodical, co- and counter-current extraction of substance from solid porous particles [1, 2] and the problem of determination of the temperature field of a solid particle heated by gases at co- and counter-current [3, 5] are fully identical to the problem of heating a body in a bounded volume of a well-mixing liquid.

The most general solution of the problem is given in [7] where identical particles of arbitrary finite form are considered, subject to arbitrary initial distributions and arbitrary source functions. From this solution one can easily obtain as special cases the solutions given in [1–10].

But in real processes the particles have different forms and sizes. In [1, 2] it is shown that at the beginning of the process the concentrations in the finer fractions considerably decrease, becoming lower than the equilibrium concentration of the mixture. As time goes on and depending on the increase of the concentration of the extracting liquid the fine fractions stop to give substance away and later begin to absorb it from the surrounding liquid up to the time when equilibrium concentration is reached. The larger particles behave in quite a different manner—in them concentration continuously diminishes down to the equilibrium one [1, 2].

It is the purpose of the present paper to render a general analytical solution for the determination of the concentration field of q different particles of arbitrary finite form, which are being placed in a bounded volume of extracting liquid. From a mathematical point of view the results obtained represent a new step in the development of the theory of finite integral transforms, given in [11–13].

STATEMENT AND SOLUTION OF THE PROBLEM

The problem may be expressed in general form by the following partial differential equations

$$w_m(M)\frac{\partial T_m(M,\tau)}{\partial \tau} = \operatorname{div}[k_m(M)\operatorname{grad} T_m(M,\tau)] + P_m(M,\tau),$$
$$m = 1, 2, \ldots, q,\ M \in V_m, \qquad \tau > 0 \tag{1}$$

with the boundary and initial conditions

$$A_m \frac{\partial T_m(N,\tau)}{\partial n} + B_m T_m(N,\tau) = \varphi(\tau), \quad m = 1, 2, \ldots, q,\ N \in S_m, \quad B_m \neq 0 \tag{2}$$

$$\frac{d\varphi(\tau)}{d\tau} + \sum_{m=1}^{q} C_m \int_{S_m} k_m(N)\frac{\partial T_m(N,\tau)}{\partial n}\, dS_m = Q(\tau) \tag{3}$$

$$T_m(M,0) = f_m(M), \qquad \varphi(0) = \varphi_0. \tag{4}$$

The mass balance conditions (3) by means of liquid phase potential $\varphi(\tau)$ couples the unknown unsteady distributions $T_m(M,\tau)$.

If $\varphi(\tau)$ is assumed to be a prescribed function of the time variable τ the desired solution can be obtained as a special case of the problem treated in [11]. After substituting the results into equation (3) one gets an integral equation for the determination of $\varphi(\tau)$, which may be solved numerically. This method leads to computational difficulties and hence it will be simultaneously examined with equations (1)–(4).

For the solution of the problem it is convenient to start applying the well known Laplace Transform with parameter p on equations (1)–(4).

$$p w_m(M) T_m(M,p) = w_m(M) f_m(M) + \operatorname{div}[k_m(M)\operatorname{grad} T_m(M,p)] + P_m(M,p) \tag{5}$$

$$A_m \frac{\partial T_m(N,p)}{\partial n} + B_m T_m(N,p) = \varphi(p) \tag{6}$$

$$p\varphi(p) + \sum_{m=1}^{q} C_m \int_{S_m} k_m(N)\frac{\partial T_m(N,p)}{\partial n}\, dS_m = \varphi_0 + Q(p). \tag{7}$$

After integrating in V_m equation (5) and substituting the result in equation (7) one finds the formula for $\varphi(p)$. Then equation (6) may be written as follows

$$A_m \frac{\partial T_m(N,p)}{\partial n} + B_m T_m(N,p) + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) T_m(M,p)\, dV_m$$
$$= \frac{1}{p}\left\{\varphi_0 + Q(p) + \sum_{m=1}^{q} C_m \left[\int_{V_m} w_m(M) f_m(M)\, dV_m + \int_{V_m} P_m(M,p)\, dV_m\right]\right\}. \tag{8}$$

To solve equations (5) at conditions (8) it is supposed that the generalized q—region Sturm–Liuville problem

$$\operatorname{div}[k_m(M)\operatorname{grad}\psi_{mi}(M)] + \mu_i^2 w_m(M)\psi_{mi}(M) = 0, \quad m = 1, 2, \ldots, q; \quad i = 1, 2, \ldots. \tag{9}$$

$$A_m \frac{\partial \psi_{mi}(N)}{\partial n} + B_m \psi_{mi}(N) + \sum_{i=1}^{q} C_m \int_{V_m} w_m(M)\psi_{mi}(M)\, dV_m = 0 \tag{10}$$

is granted for known.

Consider the expansions of prescribed functions $F_m(M)$ into a series

$$F_m(M) = \sum_{i=1}^{\infty} D_i \psi_{mi}(M). \tag{11}$$

Equations (9)–(10) lead to an eigenvalue problem having a common set of eigenvalues but different eigenfunctions. The problem does not belong to the conventional Sturm–Liouville family, and therefore, for the determination of the coefficients D_i it is appropriate to derive an integral condition to serve as an orthogonality relation.

Multiplying equation (9) by $\psi_{m,j}(M)$ and the same equations for the case $i = j$ by $\psi_{mi}(M)$, subtracting and integrating in V_m the resulting expressions, one gets [11]

$$(\mu_i^2 - \mu_j^2) \int_{V_m} w_m(M) \psi_{mi}(M) \psi_{mj}(M) \, dV_m = \int_{S_m} k_m(N) \begin{vmatrix} \psi_{mi}(N) & \dfrac{\partial \psi_{mi}(N)}{\partial n} \\ \psi_{mj}(N) & \dfrac{\partial \psi_{mj}(N)}{\partial n} \end{vmatrix} dS_m. \tag{12}$$

From the boundary conditions (10) $\psi_m(N)$ is determined. Substituting this result in equation (12) and using the formulae obtained after integrating equation (9) in V_m, the desired orthogonality relation is obtained as follows

$$\sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \left\{ B_m \psi_{mj}(M) + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mj}(M) \, dV_m \right\} dV_m = 0 \qquad i \neq j. \tag{13}$$

Multiplying equation (11) by

$$C_m w_m(M) \left\{ B_m \psi_{mj}(M) + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mj}(M) \, dV_m \right\}$$

and adding from $m = 1$ to q one finds the expression for D_i. Then (11) has the form

$$F_m(M) = \sum_{i=1}^{\infty} G_i \psi_{mi}(M) \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) F_m(M) \left\{ B_m \psi_{mi}(M) + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \, dV_m \right\} dV_m \tag{14}$$

where

$$G_i^{-1} = \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \left\{ B_m \psi_{mi}(M) + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \, dV_m \right\} dV_m. \tag{15}$$

For the case $F_m(M) = 1/B_m$ from (14) one gets

$$\sum_{i=1}^{\infty} G_i \psi_{mi}(M) \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \, dV_m = \left\{ B_m + B_m \sum_{m=1}^{q} \frac{C_m}{B_m} \int_{V_m} w_m(M) \, dV_m \right\}^{-1}. \tag{16}$$

To solve the problem we define the new finite integral transform

$$\tilde{T}_i(p) = \sum_{m=1}^{q} C_m B_m \int_{V_m} w_m(M) \psi_{mi}(M) T_m(M, p) \, dV_m + \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) \psi_{mi}(M) \, dV_m \cdot \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) T_m(M, p) \, dV_m. \tag{17}$$

The inversion formulae are determined by comparing equations (14) and (17) and can be written as

$$T_m(M, p) = \sum_{i=1}^{\infty} G_i \psi_{mi}(M) \tilde{T}_i(p). \tag{18}$$

Now, attention may be directed to the solving (5) at condition (8). After multiplying equations (5) and (9) by $w_m(M)\psi_{mi}(M)$ and $T_m(M, p)$ respectively, adding and integrating in V_m the results obtained, one gets

$$(p + \mu_i^2) \int_{V_m} w_m(M) \psi_{mi}(M) T_m(M, p) \, dV_m = \int_{V_m} w_m(M) \psi_{mi}(M) f_m(M) \, dV_m + \int_{S_m} k_m(N) \begin{vmatrix} \psi_{mi}(N) & \dfrac{\partial \psi_{mi}(N)}{\partial n} \\ T_m(N, p) & \dfrac{\partial T_m(N, p)}{\partial n} \end{vmatrix} dS_m + \int_{V_m} \psi_{mi}(M) P_m(M, p) \, dV_m. \tag{19}$$

Substituting this result in (17) we obtain

$$\tilde{T}_i(p) = \frac{1}{p+\mu_i^2}\sum_{m=1}^{q} C_m B_m \left\{ \int_{V_m} w_m(M)\psi_{mi}(M) f_m(M)\,\mathrm{d}V_m + \int_{V_m} \psi_{mi}(M) P_m(M,p)\,\mathrm{d}V_m \right\}$$

$$+\frac{1}{p+\mu_i^2}\sum_{m=1}^{q} C_m \int_{S_m} k_m(N) \begin{vmatrix} B_m\psi_{mi}(N) & \dfrac{\partial\psi_{mi}(N)}{\partial n} \\ B_m T_m(N,p) & \dfrac{\partial T_m(N,p)}{\partial n} \end{vmatrix} \mathrm{d}S_m$$

$$+\sum_{m=1}^{q} C_m \int_{V_m} w_m(M)\psi_{mi}(M)\,\mathrm{d}V_m \cdot \sum_{m=1}^{q} C_m \int_{V_m} w_m(M) T_m(M,p)\,\mathrm{d}V_m. \tag{20}$$

From the boundary conditions (8) and (10) $B_m\psi_{mi}(N)$ and $B_m T_m(N,p)$ are determined. Substituting this result in (20) and using formulae obtained after integrating (5) and (9) in V_m, one gets

$$\tilde{T}_i(p) = 1/p\left\{\varphi_0 + Q(p) + \sum_{m=1}^{q} C_m \left| \int_{V_m} w_m(M) f_m(M)\,\mathrm{d}V_m + \int_{V_m} P(M,p)\,\mathrm{d}V_m \right|\right\}$$

$$\times \sum_{m=1}^{q} C_m \int_{V_m} w_m(M)\psi_{mi}(M)\,\mathrm{d}V_m + \frac{1}{p+\mu_i^2}\left\{ \sum_{m=1}^{q} B_m C_m \left| \int_{V_m} w_m(M)\psi_{mi}(M) f_m(M)\,\mathrm{d}V_m \right.\right.$$

$$\left.\left. + \int_{V_m} \psi_{mi}(M) P_m(M,p)\,\mathrm{d}V_m \right| - [\varphi_0 + Q(p)] \sum_{m=1}^{q} C_m \int_{V_m} w_m(M)\psi_{mi}(M)\,\mathrm{d}V_m \right\} \tag{21}$$

Substituting this solution in the inversion formulae (18), after taking into account (16) and using the inverse Laplace transform the desired solutions are obtained as follows

$$T_m(M,\tau) = \left\{ B_m + B_m \sum_{m=1}^{q} \frac{C_m}{B_m} \int_{V_m} w_m(M)\,\mathrm{d}V_m \right\}^{-1} \left\{ \varphi_0 + \int_0^{\tau} Q(\tau^*)\,\mathrm{d}\tau^* \right.$$

$$+ \sum_{m=1}^{q} C_m \left| \int_{V_m} w_m(M) f_m(M)\,\mathrm{d}V_m + \int_0^{\tau}\int_{V_m} P_m(M,\tau^*)\,\mathrm{d}V_m\,\mathrm{d}\tau^* \right|$$

$$+ \sum_{i=1}^{\infty} G_i \psi_{mi}(M)\exp(-\mu_i^2\tau) \left\{ \sum_{m=1}^{q} B_m C_m \left| \int_{V_m} w_m(M)\psi_{mi}(M) f_m(M)\,\mathrm{d}V_m \right.\right.$$

$$\left. + \int_0^{\tau} \exp(\mu_i^2\tau^*)\psi_{mi}(\mathrm{M})\mathrm{P}_m(\mathrm{M},\tau^*)\,\mathrm{d}V_m\,\mathrm{d}\tau^* \right|$$

$$\left. - \left| \varphi_0 + \int_0^{\tau} \exp(\mu_i^2\tau^*) Q(\tau^*)\,\mathrm{d}\tau^* \right| \sum_{m=1}^{q} C_m \int_{V_m} w_m(M)\psi_{mi}(M)\,\mathrm{d}V_m \right\}. \tag{22}$$

It is important to point out that if parts of the surfaces S_{0m} of the particles are insulated, that is if $\partial T_m(N_0,\tau)/\partial n = 0$, $N_0 \in S_{0m}$, than the solution (22) is still valid, but of course one has to take into account the condition $\partial\psi_{mi}(N_0)/\partial n = 0$ when determining the eigenvalues and eigenfunctions from equations (9)–(10).

ONE-DIMENSIONAL SOLUTION

As an application of the general theory consider the one-dimensional case, described by the differential equations system

$$w_m(x)\frac{\partial T_m(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left\{ k_m(x)\frac{\partial T_m(x,\tau)}{\partial x} \right\} + P_m(x,\tau) \quad m = 1,2,\ldots,q, \quad x_0 \leqslant x \leqslant x_1, \quad \tau \geqslant 0 \tag{23}$$

at the following boundary

$$\frac{\partial T_m(x_0,\tau)}{\partial x} = 0, \quad A_m\frac{\partial T_m(x_1,\tau)}{\partial x} + B_m T_m(x_1,\tau) = \varphi(\tau) \tag{24}$$

$$\frac{\mathrm{d}\varphi(\tau)}{\mathrm{d}\tau} + \sum_{m=1}^{q} C_m k_m(x_1)\frac{\partial T_m(x_1,\tau)}{\partial x} = Q(\tau) \tag{25}$$

and initial conditions

$$T_m(x,0) = f_m(x), \quad \varphi(0) = \varphi_0. \tag{26}$$

Equations (9) and (10) giving the eigenvalues and eigenfunctions take the form

$$\frac{d}{dx}\left| k_m(x)\frac{d\psi_{mi}(x)}{dx}\right| + \mu_i^2 w_m(x)\psi_{mi}(x) = 0 \tag{27}$$

$$\frac{d\psi_{mi}(x_0)}{dx} = 0 \tag{28}$$

$$A_m\frac{d\psi_{mi}(x_1)}{dx} + B_m\psi_{mi}(x_1) + \sum_{m=1}^{q} C_m\int_{x_0}^{x_1} w_m(x)\psi_{mi}(x)\,dx = 0. \tag{29}$$

The solutions (22) for the one-dimensional case will be

$$\begin{aligned} T_m(x,\tau) = {} & \left(B_m + B_m\sum_{m=1}^{q}\frac{C_m}{B_m}\int_{x_0}^{x_1} w_m(x)\,dx\right)^{-1}\Bigg\{\varphi_0 + \int_0^{\tau} Q(\tau^*)\,d\tau^* \\ & + \sum_{m=1}^{q} C_m\int_{x_0}^{x_1}\left| w_m(x)f_m(x) + \int_0^{\tau} P_m(x,\tau^*)\,d\tau^*\right| dx\Bigg\} \\ & + \sum_{i=1}^{\infty} G_i\psi_{mi}(x)\exp(-\mu_i^2\tau)\Bigg\{\sum_{m=1}^{q} B_m C_m\int_{x_0}^{x_1}\psi_{mi}(x) \\ & \times\left| w_m(x)f_m(x) + \int_0^{\tau}\exp(\mu_i^2\tau^*)P_m(x,\tau^*)\,d\tau^*\right| dx \\ & - \left|\varphi_0 + \int_0^{\tau}\exp(\mu_i^2\tau^*)Q(\tau^*)\,d\tau^*\right|\sum_{m=1}^{q} C_m\int_{x_0}^{x_1} w_m(x)\psi_{mi}(x)\,dx\Bigg\} \end{aligned} \tag{30}$$

where

$$G_i = \left\{\sum_{m=1}^{q} C_m\int_{x_0}^{x_1} w_m(x)\psi_{mi}(x)\left| B_m\psi_{mi}(x) + \sum_{m=1}^{q} C_m\int_{x_0}^{x_1} w_m(x)\psi_{mi}(x)\,dx\right| dx\right\}^{-1}. \tag{31}$$

At the end let us consider an extraction of a mixture of particles, which have the form of a plate ($\Gamma_m = 0$), cylinder ($\Gamma_m = 1$) and sphere ($\Gamma_m = 2$). The extraction is defined by the following mathematical model:

$$\omega_m^2\xi^{\Gamma_m}\frac{\partial\theta_m(\xi, Fo)}{\partial Fo} = \frac{\partial}{\partial\xi}\left\{\xi^{\Gamma_m}\frac{\partial\theta_m(\xi, Fo)}{\partial\xi}\right\}$$

$$m = 1, 2, \ldots, q, \quad 0 \leqslant \xi \leqslant 1, \quad Fo \geqslant 0 \tag{32}$$

$$\frac{\partial\theta_m(0, Fo)}{\partial\xi} = 0, \qquad \frac{1}{Bi_m}\frac{\partial\theta_m(1, Fo)}{\partial\xi} + \theta_m(1, Fo) = \theta_f(Fo) \tag{33}$$

$$\frac{d\theta_f(Fo)}{dFo} + \sum_{m=1}^{q} K_m\frac{\partial\theta_m(1, Fo)}{\partial\xi} = 0 \tag{34}$$

$$\theta_m(\xi, 0) = f_m(\xi), \qquad \theta_f(0) = \theta_0. \tag{35}$$

In equation (32) the factor of the form Γ_m has a subscript m, which allows for every fraction of particles the corresponding geometry to be chosen.

The problem defined in (32)–(35) is a particular case of (23)–(26): $x = \xi$, $\tau = Fo$, $T_m(x,\tau) = \theta_m(\xi, Fo)$, $w_m(x) = \omega_m^2\xi^{\Gamma_m}$, $k_m(x) = \xi^{\Gamma_m}$, $x_0 = 0$, $x_1 = 1$, $A_m = 1/Bi_m$, $B_m = 1$, $\varphi(\tau) = \theta_f(Fo)$, $\varphi_0 = \theta_0$, $C_m = K_m$, $Q(\tau) = 0$.

For this case the solution can be given through the functions

$$W_{\Gamma_m}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!!(\Gamma_m + 2n - 1)!!} \tag{36}$$

$$V_{\Gamma_m}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n)!!(\Gamma_m + 2n + 1)!!}. \tag{37}$$

The properties of $W_{\Gamma_m}(x)$ and $V_{\Gamma_m}(x)$ are described in detail in the monography [6] and partially in the Appendix of [14]. For $\Gamma_m = 0$, 1 and 2 the series (36)–(37) define the following well-known functions

$$W_0(x) = \cos x, \quad W_1(x) = J_0(x), \quad W_2(x) = \sin x/y$$

$$V_0(x) = \sin x, \quad V_1(x) = J_1(x), \quad V_2(x) = (\sin x - x\cos x)/x^2. \tag{38}$$

The solution of (27) for $k_m(x) = \xi^{\Gamma_m}$ and $w_m(x) = \omega_m^2 \xi^{\Gamma_m}$, and having in mind the boundary condition (28), has the form

$$\psi_{mi}(\xi) = E_m W_{\Gamma_m}(\omega_m \mu_i \xi) \tag{39}$$

where E_m are integration constants.

Substituting this solution in the boundary condition (29) yields the following transcendental equation for the calculation of the eigenvalues μ_i $(i = 1, 2, \ldots)$:

$$\sum_{m=1}^{q} K_m \omega_m/\mu_i(\omega_m \mu_i/Bi_m - W_{\Gamma_m}(\omega_m \mu_i)/V_{\Gamma_m}(\omega_m \mu_i))^{-1} = 1. \tag{40}$$

Substitution of (39) in (30) after some simple algebraic transformations leads to the following analytical solution:

$$\theta_m(\xi, Fo) = \left(1 + \sum_{m=1}^{q} K_m \omega_m^2/(\Gamma_m + 1)\right)^{-1} \left(\theta_0 + \sum_{m=1}^{q} K_m \omega_m^2 \int_0^1 \xi^{\Gamma_m} f_m(\xi)\,d\xi\right)$$
$$+ \sum_{i=1}^{\infty} A_{mi} W_{\Gamma_m}(\omega_m \mu_i \xi) \exp(-\mu_i^2 Fo)$$
$$\times \left(\theta_0 + \sum_{m=1}^{q} K_m \omega_m^2 \int_0^1 \xi^{\Gamma_m} W_{\Gamma_m}(\omega_m \mu_i \xi) f_m(\xi)\,d\xi \Big/ \left(W_{\Gamma_m}(\omega_m \mu_i) - \frac{\omega_m \mu_i}{Bi_m} V_{\Gamma_m}(\omega_m \mu_i)\right)\right) \tag{41}$$

where

$$A_{mi} = \left\{\left(W_{\Gamma_m}(\omega_m \mu_i) - \frac{\omega_m \mu_i}{Bi_m} V_{\Gamma_m}(\omega_m \mu_i)\right)\left(1 + 1/2 \sum_{m=1}^{q} K_m \omega_m^2 (W_{\Gamma_m}^2(\omega_m \mu_i))\right.\right.$$
$$\left.\left. + V_{\Gamma_m}^2(\omega_m \mu_i) + (1 - \Gamma_m) W_{\Gamma_m}(\omega_m \mu_i) \frac{V_{\Gamma_m}(\omega_m \mu_i)}{\omega_m \mu_i}\right) \Big/ \left| W_{\Gamma_m}(\omega_m \mu_i) - \frac{\omega_m \mu_i}{Bi_m} V_{\Gamma_m}(\omega_m \mu_i)\right|^2 \right\}^{-1}. \tag{42}$$

CONCLUSIONS

The above solutions from the present paper permits one to solve easy any particular case of solid–liquid extraction problems. The general problem treated by the author in [7] becomes a special case of the problem studied here, namely: $q = 1$, $w_m(M) = 1$, $k_m(M) = a$, $P_m(M, \tau) = w(M, \tau)/(c\gamma)$, $A_m = \lambda/\alpha$, $B = 1$, $\varphi_0 = T_c$, $Q(\tau) = 0$, $C_m = c\gamma/(c_f \gamma_f V_f)$. Similarly, from equation (41) one can easily obtain the one-dimensional solutions in the papers [1–10].

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SOLUTION GENERÀLE DU MECANISME DE DIFFUSION EN EXTRACTION SOLIDE–LIQUIDE

Résumé—On donne une solution analytique pour les distributions de concentration en q particules différentes pour des géométries quelconques et des conditions initiale arbitraires, dans des volumes finis. La variation de concentration du liquide environnant est couplée à la séparation de matière en provenance des particules poreuses par les conditions de bilan massique pour la phase liquide. La solution obtenue contient comme cas très particuliers le problème de l'extraction périodique à co/ou/contrecourant de particules monodimensionnelles (plaque, cylindre et sphère) [1, 2, 8, 9, 10], le problème des distributions de température dans une particule solide chauffée par des gaz à co/ou/contrecourant [3, 5] et le problème du chauffage d'un corps dans un volume fini de liquide parfaitement mélangé [4, 7].

EINE ALLGEMEINE LÖSUNG FÜR DIFFUSIONSVORGÄNGE BEI DER FEST-FLÜSSIG-EXTRAKTION

Zusammenfassung—Für die Konzentrationsverteilungen in q verschiedenen Partikeln willkürlicher endlicher Geometrie und bei willkürlichen Anfangsbedingungen wird eine analytische Lösung für den Fall der Extraktion in endlichem Volumen abgeleitet. Die Konzentrationsveränderung der umgebenden Flüssigkeit hängt mit der Stoffabtrennung von den porösen Partikeln bei den für die Flüssigkeitsphase gültigen Stoffaustauschbedingungen zusammen. In der gefundenen Lösung sind die periodische Extraktion und die Extraktion bei Gleichund Gegenstrom von eindimensionalen Partikeln (Platte, Zylinder, Kugel) als Spezialfälle enthalten [1, 2, 8–10]. Zusätzlich kann die Temperaturverteilung in gasbeheizten Feststoffpartikeln bei Gleich- und Gegenstrom [3, 5] und die Erwärmung eines Körpers in einer gutdurchmischten Flüssigkeit begrenzten Volumens [4, 7] ermittelt werden.